Modal Logics Between $\mathbf{K}^3$ and $\mathbf{S5}^3$

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Given Kripke frames (W_0, R_0), (W_1, R_1) and (W_2, R_2), their *product* is defined to be the relational structure $(W_0 \times W_1 \times W_2, \bar{R}_0, \bar{R}_1, \bar{R}_2)$, where

$$(x_0, x_1, x_2)\bar{R}_\ell(y_0, y_1, y_2) \quad \text{iff} \quad x_\ell R_\ell y_\ell \text{ and } x_j = y_j \text{ for } j \neq \ell.$$

$\mathbf{K}^3$ and $\mathbf{S5}^3$ are the 3-modal logics (i.e., sets of formulas of the language having the Booleans and three diamonds) determined by all (ternary) product frames and all products of **S5**-frames, respectively.

Theorem. *Every 3-modal logic between $\mathbf{K}^3$ and $\mathbf{S5}^3$ is undecidable, non-finitely axiomatizable, and lacks the product finite model property. There is no algorithm deciding whether a finite frame validates the logic.*

This answers questions of Gabbay–Shehtman [1]. Thus, $\mathbf{K}^3$ is a natural example of an undecidable, recursively enumerable, Kripke complete logic which has the finite model property (for the latter, see [1]).

In the proofs we use the following result of Hirsch–Hodkinson [2]: It is undecidable whether a finite relation algebra is representable.

[1] D.M. Gabbay and V.B. Shehtman. Products of modal logics, part I. *Logic Journal of the IGPL*, 6:73–146, 1998.

[2] R. Hirsch and I. Hodkinson. Representability is not decidable for finite relation algebras. *Trans. Amer. Math. Soc.*, to appear.

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