On the Approximation Relation on Dynamic Logic Models

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We deal with the equivalence problem for deterministic propositional sequential programs whose semantics is defined by means of dynamic logic models. Let $\mathcal{A}$ and $\mathcal{P}$ be the alphabets of basic actions and propositional variables. A sequential program π over $\mathcal{A}$ and $\mathcal{P}$ is a finite transition system whose nodes are associated with basic actions and transitions are labelled by formulae built of propositional variables by means of connectives $\neg$ and $\wedge$. Dynamic logic models over $\mathcal{A}$ and $\mathcal{P}$ are defined as in [1]. We assume that some initial state s_0 is distinguished in every model σ. The run $\pi(\sigma)$ of a program π on a model σ is defined as usual (see [2]). If $\pi(\sigma)$ terminates and reaches some state s in σ then the result $[\pi(\sigma)]$ of this run is s; otherwise $[\pi(\sigma)]$ is undefined. By a *semantics* we mean any set M of dynamic logic models. If all models in M are based on the same frame $\mathcal{F}$ then M is called a *homogeneous* semantics. Programs π_1 and π_2 are called *equivalent* on M ($\pi_1 \sim_M \pi_2$ in symbols) iff $[\pi_1(\sigma)] = [\pi_2(\sigma)]$ holds for each model σ in M. We say that a semantics M_1 *approximates* semantics M_2 ($M_2 \preceq M_1$) iff $\pi_1 \sim_{M_1} \pi_2$ implies $\pi_1 \sim_{M_2} \pi_2$ for every pair of programs π_1, π_2. If both $M_2 \preceq M_1$ and $M_1 \preceq M_2$ hold then we write $M_1 \approx M_2$. A semantics M is called *saturated* iff $M \not\approx M \cup \{\sigma\}$ holds for every model σ which is not in M. A homogeneous semantics M based on a frame $\mathcal{F}$ is called *homogeneously saturated* iff $M \not\approx M \cup \{\sigma\}$ holds for every model σ based on $\mathcal{F}$ which is not in M. We study the lattice-theoretic properties of semantics w.r.t. approximation relation $\preceq$. Denote by $AP(M_1, M_2)$ the following assertion: each model σ_2 in M_2 is a homomorphic image of some model σ_1 in M_1. Clearly, $AP(M_1, M_2)$ implies $M_2 \preceq M_1$ for arbitrary semantics.

Proposition 1. Suppose M_1 and M_2 are saturated semantics. Then $M_2 \preceq M_1$ iff $AP(M_1, M_2)$ holds.

Corollary 1. The set of saturated semantics is a complete lattice w.r.t. $\preceq$.

Proposition 2. There exists a pair M_1, M_2 of homogeneously saturated semantics such that $M_2 \preceq M_1$ but $AP(M_1, M_2)$ does not hold.

Corollary 2. The set of homogeneously saturated semantics is a lower semi-lattice but not a lattice.

Corollary 3. There exists a saturated semantics M which has not the least homogeneous approximation.

[1] Harel D., Dynamic logics // Handbook of Philosophical Logics, D.Gabbay and F.Guenthner (eds.), 1984, p.497-604.

[2] Zakharov V.A. An efficient and unified approach to the decidability of equivalence of propositional programs // Lecture Notes in Computer Science, 1998, **1443**, p.247-259.

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