Provability Logics with Quantifiers on Proofs

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The language qLP of the provability logic with quantifiers on proofs contains propositional letters, proof variables, boolean connectives, quantifiers over the proof variables and binary proof operator $x : F$ ("x is a proof of the formula F") introduced by S.N.Artemov in [1].

This language naturally extends the language of the logic of proofs considered in [1]. The operational properties of provability studied in [2] could be expressed in this language by the corresponding quantified sentences:

$$\forall x \forall y \exists z (x : (A \to B) \to (y : A \to z : B)),$$
$$\forall x \exists y (x : A \to y : (x : A)), \text{ etc.}$$

Also, this language may be considered as an extension of the propositional modal language, since the modal provability operator $\Box A$ could be expressed by the formula $\exists x (x : A)$.

An arithmetical interpretation $*$ assigns arithmetical sentences to propositional variables, commutes with boolean connectives and quantifiers and $(x : A)^* = Prf(x, \ulcorner A^* \urcorner)$. (We suppose that the set of proof variables coincides with the set of individual variables in arithmetic.)

Given an arithmetical theory T and a class of proof predicates K we define the logic $qLP_K(T)$ as the set of all formulas in the language qLP, which are provable in T under every arithmetical interpretation $*$ based on proof predicates from K.

We present recent results concerning the decidability and arithmetical complexity of the corresponding logics for different T and K. It turned out that for many natural classes K the logic $qLP_K(PA)$ is Π^0_2-hard. For the case of the Gödel proof predicate decidable fragments were found.

[1] S. Artemov, *Logic of Proofs.* APAL, **67** (1994), pp. 29–59.
[2] S. Artemov, *Explicit provability: the intended semantics for intuitionistic and modal logic.* Techn. Rep. 98–10. MSI, Cornell University, 1998.

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