The Continuum Hypothesis

W. Hugh Woodin

TUTORIAL

I shall survey in these three lectures some recent mathematical developments which support the claim that the Continuum Hypothesis is false. The approach is to investigate abstractly the possibilities for "good" theories for the structure of the set of all subsets of ω_1, the least uncountable ordinal. This structure is simply the structure $H(\omega_2)$ of sets of hereditary cardinality at most $\aleph_1$. Informally a theory is "good" if it is finitely axiomatizable modulo consistency statements. More formally, a theory is good if it is finitely axiomatizable, over ZFC, in Ω-logic. Ω-logic is a specific strong logic which is immune to forcing. The Ω-Conjecture asserts that Ω-logic is the strongest such logic. The validities of Ω-logic define abstractly a consistency hierarchy which the Ω-Conjecture implies is cofinal among all large cardinal axioms. There are good theories for $H(\omega_2)$, but any good theory must yield that CH is false. At the core of these results are the combinatorics associated to the stationary, co-stationary, subsets of ω_1. Indeed these sets play the pivotal role.

Address: Department of Mathematics, University of California, Berkeley, CA 94720-3840, USA
E-mail: woodin@math.berkeley.edu