Liouville Functions

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A Liouville NUMBER is an irrational real number that can be well approximated by rational numbers to an accuracy that is just sufficient for Liouville's theorem (on the lack of good rational approximation to algebraic numbers) to imply that it must be transcendental. By analogy,I define a Liouville FUNCTION to be an entire,non-polynomial function that can be very well approximated by polynomials with rational coefficients. The precise degree of accuracy is defined in terms of a growth condition on the heights of the (rational) Taylor coefficients of the function,and is just sufficient to allow application of effective quantifier elimination (for the complex field) and of effective versions of Hilbert's Nullstellensatz in order to show that no hidden relationships hold between values of terms built up from the function using algebraic operations (the 'Schanuel property').On the other hand,all "generic" sets of terms (with integer coefficients)do have zeros. This allows us to apply recent results of Zilber,based on Hrushovski's amalgamation method,to conclude that certain substructures of the expansion of the complex field by a Liouville function have nice model-theoretic properties,eg omega-stability and almost model completeness.

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