On Unary Functions Definable in Coset-Minimal Ordered Groups

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A totally ordered group G is *coset-minimal* [1] if every definable subset of G is a finite union of cosets of definable subgroups intersected with intervals with endpoints in $G \cup \{-\infty, +\infty\}$. We study definable unary functions in coset-minimal ordered groups. Main results: Every unary definable function in a discrete coset-minimal ordered group is piecewise linear. Every unary definable function in a dense coset-minimal ordered group is piecewise monotone. Let G be dense and $\mathcal{F}$ the family of all definable unary functions $f : G_1 \to G_1$, where $G_1 \leq G$ is convex definable and o-minimal and G/G_1 is discrete. Then any G-definable unary function is definable in the reduct to the language $\{<, +, \mathcal{F}\}$. Dense coset-minimal ordered groups have the Exchange Property.

Definition 1 [1] Let f be a definable function from a group G to a group H. Then f is *piecewise linear* if there is a finite partition of G into definable sets $\{X_i : i \in I\}$, elements $g_i \in G$ and $h_i \in H$ and homomorphisms $\sigma_i : \langle X_i - g_i \rangle \to H$ for all $i \in I$, such that $f(x + g_i) - h_i$ is the restriction of σ_i to $X_i - g_i$ for all $i \in I$.

[1] F. Point and F. Wagner, *Essentially periodic ordered groups*, to appear in: **Communications in Algebra**.

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