Realization of Intuitionistic KPM in Explicit Mathematics

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We continue the study of realizability of different intuitionistic theories in Feferman's Explicit Mathematics began in [1] and [2]. It's easily seen that intuitionistic Kripke-Platek Set Theory **IKP** is a subsystem of Aczel's Constructive Set Theory **CZF**, so that embedding of **IKP** into $\mathbf{T_0}$ is already contained in [2]. However, analogous relations don't hold for much stronger theories **IKPi** and **IKPM**, which are intuitionistic versions of familiar **KPi** and **KPM**, resp. Nevertheless, we can modify our realizability interpretation to take care of these theories as well.

The key question is how to realize admissibility. If we take it to be expressed by a new predicate symbol Ad, then we define $\mathfrak{r}\ \underline{\mathrm{rn}}\ Ad(\alpha) \equiv \mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}} Ad(\alpha)$ as

$$\mathsf{p}_{000}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\mathrm{Tran}[\alpha] \wedge \mathsf{p}_{001}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\, Pair^{\alpha} \wedge \mathsf{p}_{010}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\, Union^{\alpha} \wedge \mathsf{p}_{011}\mathfrak{r}\ \underline{\mathrm{rn}}^{\mathcal{E}}\, Infinity^{\alpha}\ \wedge$$
$$\forall x\,\varepsilon\,\alpha\forall\mathfrak{x}(\mathsf{ln}x = 1 \to \mathsf{p}_{100}\mathfrak{r}x\mathfrak{x}\ \underline{\mathrm{rn}}^{\mathcal{E}}(s[\mathsf{str}(\alpha,x),\mathfrak{x}] \in \alpha))\ \wedge$$
$$\forall x\,\varepsilon\,\alpha\forall\mathfrak{x}(\mathsf{ln}x = 1 \wedge \forall y\,\varepsilon\,\mathsf{str}(\alpha,x)(\mathsf{ln}y = 1 \to \mathfrak{x}y\ \varepsilon\ \alpha \wedge \mathsf{ln}(\mathfrak{x}y) = 1) \to \mathsf{p}_{101}\mathfrak{r}x\mathfrak{x}\ \underline{\mathrm{rn}}^{\mathcal{E}}(sc[\mathsf{str}(\alpha,x),\mathfrak{x}] \in \alpha)),$$

where $\mathsf{p}_{\mathsf{klm}}\mathfrak{r}$ is $\mathsf{p}_{\mathsf{k}}(\mathsf{p}_{\mathsf{l}}(\mathsf{p}_{\mathsf{m}}\mathfrak{r}))$, $\mathsf{k},\mathsf{l},\mathsf{m} = 0, 1$,

$$s[\beta,\mathfrak{x}] :\doteq \{c\ \varepsilon\ \mathsf{seq} \mid \mathsf{ln}c = 0 \vee (\mathsf{ln}c \neq 0 \wedge c = \mathsf{sm}(\mathsf{sm}_0 c, \mathfrak{x}) \wedge \mathsf{sm}_0 c\ \varepsilon\ \beta)\},$$
$$sc[\beta,x] :\doteq \{c\ \varepsilon\ \mathsf{seq} \mid \mathsf{ln}c = 0 \vee (\mathsf{ln}c \neq 0 \wedge \mathsf{head}c\ \varepsilon\ \beta \wedge \mathsf{bodyt}c\ \varepsilon\ \mathsf{j}(\bar{\beta}, \lambda y.\mathsf{str}(\alpha,\mathfrak{x}y)))\},$$
$$\bar{\beta} :\doteq \{y\ \varepsilon\ \beta \mid \mathsf{ln}y = 1\},$$

and the rest of definitions are the same as in [2]. With these definitions Ad-axioms are realized and all corresponding theorems of [2] still hold, including

Theorem 4 **IKPM** is realizable in $\mathbf{T_m}$.

[1] S. Tupailo. Realization of analysis into Explicit Mathematics. *Submitted*,
`http://www.iam.unibe.ch/~sergei/an.ps`

[2] S. Tupailo. Realization of Constructive Set Theory into Explicit Mathematics: a lower bound for impredicative Mahlo universe. *Submitted*,
`http://www.iam.unibe.ch/~sergei/czf.ps`

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