BN-pairs of Finite Morley Rank

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We report on recent progress on the Cherlin-Zil'ber Conjecture, viz. that a simple group of finite Morley rank is an algebraic group over an algebraically closed field. The conjecture implies in particular that any such group must have a definable 'split' BN-pair. The following is a partial analog of the famous paper by Fong and Seitz on finite groups with split BN-pairs:

Theorem: [T] *Let G be a simple group with a definable BN-pair of rank 2 where $B = U.T$ for $T = B \cap N$ and a normal subgroup U of B with $Z(U) \neq 1$. It was shown in [TVM2] that the Weyl group $W = N/B \cap N$ has cardinality $2n$ with $n = 3, 4, 6, 8$ or 12. If G has finite Morley rank then furthermore the following holds:*
(i) If $n = 3$, then G is definably isomorphic to $PSL_3(K)$ for some algebraically closed field K.
(ii) If U is nilpotent, then $n \neq 12$.
(iii) If $Z(U)$ contains a B-minimal subgroup A with $RM(A) \geq RM(P_i/B)$ for both parabolic subgroups P_1 and P_2, then $n = 3, 4$ or 6 and G is definably isomorphic to $PSL_3(K), PSp_4(K)$ or $G_2(K)$ for some algebraically closed field K.

Note that the conditions in (iii) are necessarily satisfied if the Cherlin-Zil'ber Conjecture is true.

[T] K. Tent, Split BN-pairs of finite Morley rank, preprint.

[TVM1] K. Tent, H. Van Maldeghem, Cherlin's Conjecture for buildings with panels of Morley rank 1, preprint.

[TVM2] K. Tent, H. Van Maldeghem, On irreducible (B,N)-pairs of rank 2, preprint.

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