Bounded Applicative Theories

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Theories with self-application form the operational core of systems of explicit mathematics, which have been introduced by Feferman in [3]. In an applicative universe of discourse, all objects may be thought of as operations or rules, which can freely be applied to each other: self-application is meaningful, though not necessarily total. It has turned out that applicative theories provide a natural framework for studying notions of abstract computability, especially from a proof-theoretic perspective, cf. [4] for a recent survey.

This contribution is concerned with the study of (unramified) *bounded* applicative systems which have a strong relationship to classes of computational complexity. We provide new applicative theories whose provably total functions coincide with the functions computable in polynomial time, polynomial space, polynomial time *and* linear space, as well as linear space. Our theories can be seen as natural applicative analogues of systems of bounded arithmetic, cf. Buss' monograph [1]. We are also interested in higher type aspects of our systems; in particular, it is shown that Cook and Urquhart's system PV^ω (cf. [2]) is directly contained in a natural applicative theory of polynomial strength.

[1] Buss, S. R. *Bounded Arithmetic*. Bibliopolis, Napoli, 1986.

[2] Cook, S. A., and Urquhart, A. Functional interpretations of feasibly constructive arithmetic. *Annals of Pure and Applied Logic 63*, 2 (1993), 103–200.

[3] Feferman, S. A language and axioms for explicit mathematics. In *Algebra and Logic*, J. Crossley, Ed., vol. 450 of *Lecture Notes in Mathematics*. Springer, Berlin, 1975, pp. 87–139.

[4] Jäger, G., Kahle, R., and Strahm, T. On applicative theories. In *Logic and Foundations of Mathematics*, A. Cantini, E. Casari, and P. Minari, Eds. Kluwer, 1999, pp. 83–92.

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