Leibnitz Style Arithmetization of Monadic Predicate Calculus with Equality

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Leibnitz's idea to translate syllogistic into arithmetic was realized in [1]. In this paper, Leibnitz style translation is extended on the whole monadic predicate calculus including equality. To build an *arithmetical* model, let $N > 1$ be an integer *without multiple factors*. Any predicate P_i is interpreted by arbitrary divisor of N (possibly 1 or N) denoted with $d(P_i)$, and any individual variable x_i is evaluated by $d(x_i)$, a *prime* divisor of N. Following the construction of a formula F, its *arithmetical statement* $\mathcal{AR}[F]$ corresponding to given evaluation will be obtained. Namely, for atomic formulas, $\mathcal{AR}[P_i(x_j)]$ is "$d(x_j)$ divides $d(P_i)$" and $\mathcal{AR}[x_i = x_j]$ is "$d(x_i) = d(x_j)$"; for a subformula G, $\mathcal{AR}[(\forall x)G]$ is "for any prime divisor d, $\mathcal{AR}^x_d[G]$" where $\mathcal{AR}^x_d$ differs from $\mathcal{AR}$ attaching d to x; finally, all propositional connectives are replaced with their non-formal analogues. If $\mathcal{AR}[F]$ is a true arithmetical sentence for any N under arbitrary evaluation, F is named *arithmetically true*.

Theorem 1 *Any formula of the monadic predicate calculus with equality is a tautology iff it is arithmetically true.*

First of all, a predicate formula is a tautology *iff* its universal closure is a tautology. Because of the decidability of the monadic predicate calculus with equality, the domains may be supposed to be finite [2, Theorem 25.1]. Any such model can be transformed in an isomorphic arithmetical model: if all elements of the domain D are $a_1, \ldots, a_n$, take an integer $N = p_1 \cdot \ldots \cdot p_n$ where $p_1, \ldots, p_n$ are *different* prime numbers. If the interpretation of P_i is $\{a_{i_1}, \ldots, a_{i_k}\} \neq \emptyset$, let $d(P_i)$ be $p_{i_1} \cdot \ldots \cdot p_{i_k}$; if the subset is empty, $d(P_i)$ is 1. The evaluation of individual variables is obvious.

[1] V. Sotirov, "Arithmetizations of syllogistic à la Leibnitz", *J. Appl. Non-Class. Logics*, **9** (1999), n. 2–3, pp. 387–405.

[2] G. Boolos, R. Jeffrey, "Computability and Logic", Cambridge Univ. Press, 3rd ed. 1989.

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