On Valued C-minimal Groups

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D. Macpherson and C. Steinhorn introduced some variants of the notion of o-minimality. One of the most interesting ones is *C-minimality*, which provides a natural setting for studying algebraically closed valued fields and some valued groups. We have gone further in the study of the structure of C-minimal valued groups, giving a partial characterization in the abelian case, and some new structure theorems about the non-abelian case.
In the abelian case we obtain the following principle: let G be an abelian valued group for which the valuation satisfies some kind of compatibility with the multiplication by any prime number p. Then G is C-minimal if and only if the expansion of the chain of valuations I by the maps induced by the multiplication by each prime number in G, and by some unary predicates controlling the cardinality of the residual structures, is o-minimal. Using this principle we can construct many new examples of abelian C-minimal valued groups.
Our main progress in the non-abelian case concerns the case where all the residual structures are infinite: we prove that the commutator function of the group induces a function from $I \times I$ to I whose partial maps are increasing, and satisfy the intermediate value property, and some "bilinear" functions between residual structures. In [1] it was shown that any C-minimal valued group $\mathbb{G}$ contains a definable abelian normal subgroup A such that G/A is of finite exponent. We can now improve this result under our assumption on residual structures: we prove then that G/A is a nilpotent group of exponent a power of a prime number p.

[1] H. D. Macpherson, C. Steinhorn, *On variants of o-minmality*, Annals of Pure and Applied Logic 79 (1996), 165-209.

[2] P. Simonetta, *Abelian C-minimal groups*, To appear in Annals of Pure and Applied Logic.

[3] P. Simonetta, *On non-abelian C-minimal groups*, in preparation.

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