Easy Proofs of Some Theorems of Ax-Kochen-Ershov

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Emulating Abraham Robinson's proof, via model completeness, that the theory of real-closed ordered fields is complete, one may transfer model completeness and completeness from the theories of the residue field and of the value group to the theory of a corresponding Henselian valued field. This approach applies in all the familiar cases: residue characteristic zero; mixed characteristic with finite ramification; algebraically maximal Kaplansky fields. Where Robinson uses real closure to verify that his test for model completeness applies, the present approach uses Henselization–or in the last case, algebraically maximal immediate algebraic extension–to show that Robinson's test applies to immediate extensions; then it applies to arbitrary extensions by a general model-theoretic argument using no valuation theory. A simple modification of the prime-model test then allows one to handle completeness as well as model completeness.

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