The Decidability of the Existential Theory of $\mathbb{F}_p[[t]]$ and Resolution of Singularities

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I will present an unpublished proof due to Jan Denef, showing how the decidability of the existential theory of $\mathbb{F}_p[[t]]$, follows from Resolution of Singularities in characteristic p. Here t is a single variable and $\mathbb{F}_p$ the p-element field. For the positive existential theory (i.e., we only have equations), this is an immediate consequence of a Theorem of Greenberg: if one can find an approximate solution of order N, then a true solution exists, where N depends only on the degree of the equations involved. For the general case one uses the fact that, if a variety V defined over $\mathbb{F}_p((t))$ has a non-singular $\mathbb{F}_p((t))$-rational point, then the $\mathbb{F}_p((t))$-rational points on V are Zariski dense at that point. If V is a projective variety, then any $\mathbb{F}_p((t))$-rational point is also $\mathbb{F}_p[[t]]$-rational, by the Valuative Criterion for Properness. This settles the problem for a non-singular system of equations and inequalities. Finally, one reduces to the non-singular case using Resolution of Singularities.

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