Groups Definable in Compact Complex Manifolds

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A compact complex manifold considered as a first-order structure with the closed analytic subvarieties of its Cartesian powers as the basic definable sets is a totally transcendental structure of finite Morley rank. We classify the groups interpretable in compact complex manifolds. Our main results are the following theorems.

Theorem 1 *A complex Lie group G is definable in a compact complex manifold if and only if it has the structure of a meromorphic group in the sense of Fujiki: there is a compact complex manifold $\overline{G}$ into which G embeds as a Zariski dense subset and for which the group operations on G extend to meromorphic operations on $\overline{G}$ and the left and right regular actions of G on itself extend to holomorphic actions of G on $\overline{G}$.*

Theorem 2 *If G is a strongly minimal group interpretable in a compact complex manifold, then G is definably isomorphic to a one-dimensional connected algebraic group or to a non-algebraic simple complex torus.*

Theorem 3 *If G is a connected group interpretable in a compact complex manifold, then G has a definable normal subgroup $L \leq G$ which is definably isomorphic to a linear algebraic group and for which G/L is definably isomorphic to a complex torus.*

This is a report on joint work with Anand Pillay.

[1] A. Fujiki On automorphism groups of compact Kähler manifolds, *Invent. Math.* **44** (1978), no. 3, 225–258.

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