Two Preservation Thorems in Infinitary Logic

J. Rosický

A *regular* sentence in $L_{\lambda\lambda}$ is defined as $(\forall x)(\varphi(x) \to (\exists y)\psi(x,y))$ where φ and ψ are (infinitary) conjunctions of atomic formulas (and x and y are strings of variables). Regular sentences correspond to the algebraic concept of injectivity. In a case when $\exists$ is the unique existence quantifier, one speaks about *limit* sentences; they are related to the concept of orthogonality.

Theorem 1 (see [4]) *Let λ be a regular cardinal. A class of structures is axiomatized by regular sentences in $L_{\lambda\lambda}$ iff it is closed under products, λ-directed colimits and λ-pure substructures.*

Theorem 2 (see [2]) *Let λ be an uncountable regular cardinal. A class of structures is axiomatized by limit sentences in $L_{\lambda\lambda}$ iff it is closed under limits and λ-directed colimits.*

Recall that λ-pure means elementary w.r.t. positive-primitive formulas in $L_{\lambda\lambda}$ (see [1]). Any class closed under limits and λ-directed colimits is closed under products and λ-pure substructures. For $\lambda = \omega$, Theorem 1 is an easy consequence of the Compactness Theorem. Theorem 2 does not hold in this case (see [5]).

A *geometric* sentence in $L_{\infty\lambda}$ is defined as $(\forall x)(\varphi(x) \to \bigvee_{i\in I}(\exists y_i)\psi(x,y_i))$ (where φ, ψ_i, x and y_i are as above). Classes of structures axiomatized by geometric sentences are closed under λ-directed colimits and λ-pure substructures. The converse is not true even for $\lambda = \omega$ (see [4]).

The just stated results are motivated by the theory of accessible categories (see [3] and [1]) which is closely related to infinitary model theory and is useful in algebra, homotopy theory or theoretical computer science.

[1] J. Adámek and J. Rosický: Locally presentable and accessible categories *Cambridge Univ. Press 1994.*

[2] M. Hébert and J. Rosický: Uncountable orthogonality is a closure property, submitted.

[3] M. Makkai and R. Paré: Accessible categories: the foundations of categorical model theory *Contemporary Math. 104, AMS, Providence 1989*

[4] J. Rosický, J. Adámek and F. Borceux: More on injectivity in locally presentable categories, submitted

[5] H. Volger: Preservation theorems for limits of structures and global sections of sheaves of structures *Math. Z.* **166** *(1979), 27–53*

Address: Masaryk University, Janáčkovo nám. 2a, 662 95 Brno, Czech Republic
E-mail: `rosickymath.muni.cz`