Equality from the Constructive Point of View

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We consider equality in the framework of the constructive model theory developed in [1]. Let (M, ν) be an enumerated set. A constructive equality structure compatible with (M, ν) is a function $\mathcal{E} : \mathbf{N}^2 \to 2^{\mathbf{N}}$ such that $\mathcal{E}(m, n) \neq \emptyset \Leftrightarrow \nu(m) = \nu(n)$. The elements of $\mathcal{E}(m, n)$ are treated as the realizations (in the Kleene sense) of the sentence $m = n$. The set of realizations of a sentence in the language with the only predicate symbol $=$ is defined in accordance with Kleene's recursive realizability. The theory of a constructive equality structure is defined as the set of all sentences whose set of realizations is not empty.

In the classical logic, the equality theory of a set M depends only of the cardinality of M. In the constructive case the situation is rather different.

An equality structure is said to be normal if the sentence $\forall x(x = x)$ is realizable. It is proved that all normal constructive equality structures compatible with a given enumerated set have the same theory.

The next step is extracting structures with decidable equality, i. e. equality structures where the sentence $\forall x \forall y(x = y \vee \neg x = y)$ is realizable. It is proved that the theory of a normal constructive equality structure with decidable equality compatible with a given enumerated set (M, ν) is just the classical equality theory of M.

There are different constructive equality theories with undecidable equality even in the case of a 2-element enumerated set.

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[1] V. Plisko. On languages with constructive logical connectives. Dokl Akad. Nauk SSSR **296** (1987), pp. 35-38

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