A Predicate Logic Based on the Gödel Interpretation and the System of Hereditary Effective Operations

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Let T be an intuitionistic arithmetic theory, i. e. an arbitrary set of arithmetic sentences closed under derivability in the intuitionistic arithmetic HA. A closed predicate formula will be called T-valid if each its closed arithmetic instance is in T. The set of all T-valid predicate formulas will be denoted by $L(T)$. An arithmetic theory T is called an IS-theory if there are arithmetic Σ_1-formulas $A(x)$ and $B(x)$ such that the formula

$$\forall x \neg (A(x) \,\&\, B(x)) \,\&\, \neg \forall x \exists y ((A(x) \supset y = 0) \,\&\, (B(x) \supset y \neq 0))$$

is in T. An arithmetic formula is said to be negative if it does not contain $\vee$ and $\exists$. The negative fragment of a theory T is denoted by T^-.

Theorem 1 *If T is an intuitionistic IS-theory, then $T^- \leq_1 L(T)$.*

The theory T consists of arithmetic sentences A such that its Dialectica interpretation A^D in the language of arithmetic in all finite types is true if variables in A^D are interpreted as hereditary effective operations. One can to prove that T is an intuitionistic IS-theory and T^- is not recursively enumerable. The following theorem is a consequence of this fact and Theorem 1:

Theorem 2 *The logic $L(\mathsf{T})$ is not recursively enumerable.*

An analogous result for the predicate logic based on the Gödel interpretation and the system of hereditary recursive operations was proved in [1].

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[1] V. Plisko. Two semantics and logics based on the Gödel interpretation. In: Computational Logic and Proof Theory. Proc. 5th Kurt Gödel Colloquium, KGC'97, Vienna, Austria, August 1997 (Lecture Notes in Computer Science **1289**), pp. 233-240

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