Some Remarks on Games Logic Plays

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Various logical methods have recently been employed in analysing games, and conversely, game-theoretic methods have been used in evaluating logical statements. I argue that this latter connection can be made tighter than before, by describing some new game-theoretic phenomena behind some logical notions. Primarily they arise in (i) first-order formulas having non-linear (partial or non-transitive) informational independencies, traditionally corresponding to sequential games of imperfect information. (ii) One can also have more semantic dependence than in classical logic, requiring extensive form of games with cycles, while not all imperfect information games can correspond to logical languages. These considerations suggest that Henkin quantifiers should be interpreted with (genuine) simultaneous games rather than sequential games.

Some of the further connections can be described as follows. (iii) Informationally independent logics give rise to sequential games of imperfect recall (retrieval failure). (iv) Informational independence of negation can correspond to games of incomplete information (hidden chance moves). (v) Informationally independent formulas incorporate signalling and team playing, related to imperfect recall. (vi) Semantic games can be viewed as extensive form games where information sets denote (priority) scope relations, and skolemised formulas as strategic forms of games. (vii) Even linearly ordered classical logic can be seen as a logic of imperfect information and imperfect recall, as well as asymmetric in information (discriminating games). (viii) In game-theoretically interpreted modal logics, there is a need for distinguishing between two notions of information sets, suggesting a dynamisation of information sets.

(ix) All games for logic so far are strictly competitive, but not necessarily determinable, giving rise to partial logics, as a product of entirely natural assumptions about the game. (x) Games that are non-strictly competitive can amount to forms of non-coherence: by distinguishing between different notions of consistency, paraconsistencies receive game-theoretic import.

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