Open Problems Towards an Analogue of Hilbert's Tenth Problem for the Rational Numbers

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We present a - yet incomplete - effort for proving a negative answer ('undecidability') to the analogue of Hilbert's tenth problem for the field of rational, i.e. to the

Question. Is there an algorithm which, given an arbitrary diophantine equation, answers, always and correctly, whether that posseses any rational solutions?

The proposed method (some parts of which have been realized) consists of the following steps:

A. Choose a suitable elliptic curve with Mordell-Weill rank equal to one, over a suitably chosen finite extension of the rationals

B. Show that divisibility of points on the elliptic curve is existentially definable (thus produce a new proof of undecidability of the whole theory of the rationals)

C. Produce some more, existentially definable structure, which, together with addition and divisibility, gives an existential definition of 'multiplication' of points on the curve.

We present recent results and open questions testifying to the plausibility as well as the expected difficulty of the effort.

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