Wide Sets, Deep Many-Valuedness and Sorites Arguments

Carlos Pera

In this contributed talk, I show how to obtain a powerful and truthful explanation of the failure of Sorites Arguments combining an adaptation of the Wide set theory formulated by professors Formato and Gerla (1998) and the concept of deep many-valuedness established by professor Hubert Marraud (1998 and 1999). Our conclusion is that if the premises of a Sorites Argument are conceived like a succession of indexed consequence operators (where indices express the accuracy of the inferences) prefixing sentences and forming wide data sets, the failure of the Argument is because of the transitive property for the inclusion between those wide data sets is not always holded.

In a general sense, a wide set is a finite set as big as we will. Given a binary predicate EQ interpreted as a multivalued extension of the classical relation of equipotence between finite sets, and given a finite set X, Wide (X) is denoted by the formula $EQ(X, X \cup \{X\})$. An important aspect is that an infinite set can be reduced to the existence of wide sets because an infinite set is a set equipotent to its successor. On page 383 of their article, Formato and Gerla declare that sorites paradoxes are based on the notion of wide set. In fact, this is the main objective of our contributed paper: the explanation of the genesis and failure of that kind of arguments not finding out the difficulty on the gradual character of the assertion of truth of fuzzy predicates but using the theory of Formato and Gerla under the influence of the idea of deep many-valuedness developed by Marraud. Until now in the investigations about logical many-valuedness has prevailed the construction of superficial many-valued logics, that is, logics with an arbitrary number (bigger than two) of truth-values but always incorporating a bivalent consequence relation. We can wonder us why we hold the existence of logics with a number bigger than two of truth-values but in which the sentences of the kind "S_{n+1} follows from $S_1, \ldots S_n$" only can take the assignation of two truth-values. By this reason one can speak about deep many-valued logics, that is, many-valued logics whose consequence relation is $n > 2$-valued. From a deeply many-valued perspective, v.g. considering several consequence operators, the matter is not that, for example, the third premise in a sorites argument be less true (with a determined degree of truth) than the second one: it will happen that in the third premise is displayed a different consequence relation weaker than the preceding relation. So, for consequence operations C_i, a theory $C_l(D)$ is more trustworthy than a theory $C_2(D)$ iff the quantity of information dervable from a data set D is bigger under C_2, that is, C_2 is more lax than C_1.

Using tools of the Wide set theory and conditional probablities, I show that a precise explanation of the failure of sorites arguments is located on the failure of the transitivity for the inclusion between sets of premises conceived like sets of sentences prefixed by consequence operators and having indices of accuracy progressively more defeasible in order to the presence of new information added.

F. Formato and G. Gerla: 1998, 'Grasping Infinity by Finite Sets', *Mathematical Logic Quaterly* 4 4, 383-393.
H. Marraud: 1998, 'Razonamiento aproximado y grados de consecuencia', *Endoxa* 10, 55-70.
H. Marraud: 1999, 'Sorites Paradox from a Logical Point ofView', in Proc. of the Internat. Congress 'Analytic Philosophy at the Tum of the Millenium', Santiago de Compostela.
C. Pelta: 1999, 'Deep Many-Valuedness', to appear.

Address: Department of Linguistics, Logic and Ph. of Science, Universidad Autónoma de Madrid, Campus de Cantoblanco, 28049-Madrid (Spain)
E-mail: `Ivr@step.es`