Extensionality and the Fregean Axiom

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The Extensionality Principle states that coreferential expressions can be mutually interchangeable in all sentential contexts without changing their logical value, symbolically:

$$(\varphi = \phi) \Rightarrow (\alpha \Leftrightarrow \alpha[\varphi/\phi])$$

In predicate logic the Extensionality Principle is embodied in the following logical scheme

$$a = b \Rightarrow (\alpha[x/a] \Leftrightarrow \alpha[x/b])$$

In classical sentential calculus this principle translates into the following extensionality rules:

$$(\alpha \Leftrightarrow \beta) \vdash \gamma[p/\alpha] \Leftrightarrow \gamma[p/\beta], \qquad \alpha \Leftrightarrow \beta, \gamma[p/\alpha] \vdash \gamma[p/\beta]$$

These rules result from the fact that Fregean Axiom holds for classical logic. The Axiom states that the semantic correlates of sentences are their logical values. Suszko in [1], [2] rejected Fregean Axiom and following Wittgenstein assumed that the situation presented by a sentence is its semantic correlate.

To formally express the difference between the logical value of a sentence and the situation presented by it, Suszko enriched the language of classical logic by the "$\equiv$" non-truth-functional connective, which defined by means of the following logical schemes:

$\alpha_1 \equiv \alpha_2$, (where α_1, α_2 differ at most in bound variables)
$(\alpha \equiv \beta) \Rightarrow (\alpha \Leftrightarrow \beta)$,
$(\alpha \equiv \beta) \Rightarrow (\neg\alpha \equiv \neg\beta)$,
$(\alpha \equiv \beta \wedge \gamma \equiv \delta) \Rightarrow (\alpha \S \gamma \equiv \beta \S \delta)$, (where $\S : \wedge, \vee, \Rightarrow, \Leftrightarrow, \equiv$)
$\forall v(\alpha \equiv \beta) \Rightarrow (Qv\alpha \equiv Qv\beta)$, (where $Q = \forall, \exists$, and v is any variable sentential or nominal).

As a result Suszko obtained logical calculus which he called Non-Fregean Logic. This logic is a generalization of classical logic, for if we assume that "$\equiv$" = "$\Leftrightarrow$", then we obtain classical logic. Non-Fregean logic is both extensional and logically bivalent, but is not ontologically bivalent.

Non-Fregean Logic is extensional for it includes the following schemes logical theorems:

$$(\varphi \equiv \phi) \Rightarrow (\gamma[v/\varphi] \Leftrightarrow \gamma[v/\phi])$$

The calculus is logically bivalent, as it includes following logical theorems: $p \vee \neg p$, $\neg(p \wedge \neg p)$. But not ontologically bivalent as it is, an important role in it is played, unlike in classical predicate calculus, by sentential variables which run over the universe of situations.

[1] Suszko Roman, Ontology in the Tractatus of L. Wittgenstein, Notre Dame Journal of Formal Logic, 9, (1968), p.7-33.
[2] Suszko Roman, Abolition of the Fregean Axiom, Lecture Notes in Mathematics (453) (1975), p.169-239.

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