Small Profinite Groups

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Profinite groups are inverse limits of finite groups. Small profinite groups are the profinite groups interpretable in small theories.

Question. *Is every small profinite group abelian-by-finite ?*

Motivated by this question I developed some "model theory" of small profinite groups. The strongest result I proved is as follows.

Theorem. *Assume G is a small profinite group, which is m-stable. If G is solvable-by-finite, then G is nilpotent-by-finite.*

m-stability is a kind of chain condition. The proof resembles the arguments on groups of finite Morley rank.

An example of a profinite group interpretable in a theory T is the group G/G^0 for any group G definable in T. If T is superstable, then by the results of Baudisch the group G/G^0 is solvable-by-finite.

Corollary. *If T is a small superstable theory, which is m-stable, then for every group G definable in T, the group G/G^0 is nilpotent-by-finite.*

These results will appear in the paper of mine: *Small profinite groups*, to appear in J.Symb.Logic. They fit into a more general scheme. By the results of Hrushovski, Pillay and others, any group definable in an ω-stable theory with 1 countable model is abelian-by-finite. I proved the counterpart of this result for profinite groups interpretable in a superstable theory with $< 2^{\aleph_0}$ countable models. So one can wonder if the few models assumption may be weakened just to smallness. This is the motivation for the question above.

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