PCF Theory for Small Products of Arbitrary Partial Orders

Stefan Neumann

Due to Shelah, a partial order P has the true cofinality $\mathrm{tcf}(P) = \lambda$ iff λ is regular and there is a strictly increasing sequence $(p_\alpha : \alpha < \lambda)$ which is cofinal in P. Shelah's pcf theory which has applications in several areas of set theory deals with reduced products of regular cardinals and their possible true cofinalities. It is clear that its results can be applied to products of such partial orders for which the true cofinality exists since $\prod_i P_i/I$ and $\prod_i \mathrm{tcf}(P_i)/I$ are equivalent with respect to their true cofinalities.

It is natural to ask whether a similar theory can be established for products of arbitrary partial orders. Using a slightly different concept, we obtain a theory for such products which has similar structural results and the development of which is somewhat clearer and easier; e. g. the usage of ultrafilters is completely avoided. Moreover, it is easy to see that the basic results of "ordinary" pcf theory, including the so called pcf theorem, are obtained as special cases.

[1] Burke, Magidor, *Shelah's pcf Theory and its Applications*, Ann. Pure Appl. Logic 50 (1990), 207–254.

[2] Holz, Steffens, Weitz, *Introduction to Cardinal Arithmetic*, Birkhäuser, Basel, 1999.

[3] Kojman, *The A, B, C of pcf Theory*, `http://xxx.lanl.gov/`, 1995.

[4] Shelah, *Cardinal Arithmetic*, Oxford University Press, New York, 1994.

[5] Shelah, *The pcf Theorem Revisited*, in *The Mathematics of Paul Erdős*, II, 420–459, Springer, Berlin, 1997.

Address: University of Hannover, Germany