The Spectrum of a Simple Modular Theory

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In his celebrated paper [4], M. Ziegler introduced a topological space, now called the *Ziegler spectrum*, whose points are pure injective indecomposable modules. This space turned out to be rather successful in the model-theoretic investigation of modules. From the point of view of model theory, modules are more or less one-based groups (see [2]). For a stable theory that eliminates imaginaries one-basedness implies that the lattice of algebraically closed sets is modular [3].

It was recognized by B. Kim [2] that simple theories satisfy all 'algebraic properties' of forking known from stable theories. So it seems natural to ask whether there is an analogous construction for groups whose theory is simple and modular. We will construct for each such theory a topological space which coincides with the Ziegler spectrum in the module case. If furthermore the theory is supersimple, the closed subsets of the spectrum correspond to certain complete theories which allows to work on a setting that is between model theory (where one usually studies a complete theory) and representation theory (where one usually regards the whole category or certain full subcategories of modules over some specified ring).

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[2] Kim, B., Pillay, A., From stability to simplicity, The Bulletin of Symbolic Logic, Vol. 4, No. 1, 1998.

[3] Low, Lee Fong, Lattice of algebraically closed sets in one-based theories, Journal of Symbolic Logic, Vol. 59, No. 1, 1994.

[4] Ziegler, M., Model theory of modules, Ann. Pure Appl. Logic 26, 1984.

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