Some Properties of Logics with Henkin Quantifiers in Poor Vocabularies

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We consider the logic with the Henkin quantifiers L^*, for the main ideas and concepts see survey [Krynicki–Mostowski 1995]. We continue investigations of the logic L^* in empty vocabulary, see [Mostowski 1989] and [Mostowski 1991]. We prove that the set of all tautologies in infinite models of L^* in empty vocabulary is of degree $0'$. It answers the problem from [Mostowski 1991]. Moreover the same set restricted to quantifiers H_n has the same degree, what essentially follows from the result from [Krynicki–Mostowski 1992]. As a corolary we obtain that the set of all tautologies (in arbitrary models) of L^* as well as L_{H_ω} in empty vocabulary is of degree $0'$.

Additionaly, as a result, we obtain decidability of each logic $L^k(Q)$ – the logic with Henkin quantifier Q, and k variables. On the other hand – by easy generalization of [Mostowski 1989] – for each algorithm A, the statement "A decides $L^k(Q)$" cannot be proved e. g. in GB, for sufficiently large Henkin quantifier Q.

We consider also L^* in σ_n – monadic vocabulary (with n unary predicates). We prove that, assuming some natural generalization of "Chang Conjencture", for each n and each class of Henkin quantifiers A, containing at least one nonlinear Henkin quantifier, the logic $L_{\sigma_n}(A)$ either is decidable or it is not axiomatizable. This solves positively the following conjecture from [Krynicki–Mostowski 1992].

For each vocabulary σ and each class of Henkin quantifiers A, containing at least one nonlinear Henkin quantifier, the logic $L_\sigma(A)$ either is decidable or it is not axiomatizable.

Additionally we improve some undecidability results from [Krynicki–Mostowski 1992] by showing that $L_{\sigma_n}(\mathsf{H}_4)$ and $L_{\sigma_n}(\mathsf{H}_2^2)$ are undecidable for some n.

Let E_n be $\begin{matrix}\forall x \exists y_1 \ldots \exists y_n \\ \forall z \exists w_1 \ldots \exists w_n\end{matrix}$. Then we prove that pure logic with all quantifiers E_n is not decidable. It improves the results of [Krynicki–Mostowski 1992] by proving the same for the logic appearingly weaker.

[Krynicki–Mostowski 1992] M. Krynicki and M. Mostowski, *Decidability problems in language with Henkin quantifiers*, in **Annals of Pure and Applied Logic** 58 (1992), pp. 149–172.

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[Mostowski 1989] M. Mostowski, *Pure logic with branched quantifiers*, in **Zeitschrift für Mathematische Logik und Grundlagen der Mathematik** 35 (1989), pp. 45–48.

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