# A Forcing Construction for Thin-Very Tall Boolean Algebras

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A Boolean algebra $B$ is *superatomic*, if every subalgebra of $B$ is atomic. It is known that the Cantor-Bendixson process for topological spaces can be transferred to the context of Boolean algebras. If $B$ is a Boolean algebra, we define for every ordinal $\alpha$ *the ideal* $I_\alpha$ as follows: $I_0 = \{0\}$; if $\alpha = \beta + 1$, $I_\alpha =$ the ideal generated by $I_\beta \cup \{\, b \in B : b/I_\beta$ is an atom in $B/I_\beta \,\}$; and if $\alpha$ is a limit, $I_\alpha = \bigcup\{\, I_\beta : \beta < \alpha \,\}$. Then, $B$ is superatomic iff there is an ordinal $\alpha$ such that $B = I_\alpha$.

We define *the height* of a superatomic Boolean algebra $B$ by $\mathrm{ht}(B) =$ the least ordinal $\alpha$ such that $B/I_\alpha$ is finite (which means $B = I_{\alpha+1}$). For every $\alpha < \mathrm{ht}(B)$, we denote by $\mathrm{wd}_\alpha(B)$ the cardinality of the set of atoms of $B/I_\alpha$, and then we define *the width* of $B$ by $\mathrm{wd}(B) = \sup\{\, \mathrm{wd}_\alpha(B) : \alpha < \mathrm{ht}(B) \,\}$.

It was shown by Juhász and Weiss that for every ordinal $\alpha < \omega_2$, there is a superatomic Boolean algebra of width $\omega$ and height $\alpha$. Also, Baumgartner and Shelah proved that Con (ZFC) $\rightarrow$ Con (ZFC + "there is a superatomic Boolean algebra of width $\omega$ and height $\omega_2$"). Then, we will show that we can improve Baumgartner-Shelah's theorem, showing that Con (ZFC) $\rightarrow$ Con (ZFC + "for every $\alpha < \omega_3$ there is a superatomic Boolean algebra of width $\omega$ and height $\alpha$").

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