Set Theory in the 20th Century

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It is the first third of the century that I will discuss in the most detail. This was the period that witnessed the formal axiomatization of set theory. It also witnessed the discovery of the iterative concept of set, a concept that the ZFC axioms may be thought of as axiomatizing. There are still several misconceptions about this early period, particularly misconceptions having to do with credit. The contributions of Cantor and Mirimanoff, for example, are not adequately appreciated. Zermelo, on the other hand, is deservedly much appreciated, but his real contributions are not quite the same as the contributions for which he is given credit.

The relative consistency theorems of Gödel and Cohen revitalized set theory, but they also raised fundamental questions about the future of the subject. The various kinds of set-theoretic research after Cohen might be thought of as resulting from different responses to these questions. Fortunately, there are many interconnections between the different kinds of research. What is perhaps unique about set theory at the end of the century is that work in it might shed light on basic questions about the nature of mathematics. How can we come to know the truth of fundamental mathematical axioms? Do all mathematical questions have definite answers? Is such a set as the set of all real numbers a determinate totality or something ultimately vague?

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