Some Results on $\mathbf{L}\Delta^-_{n+1}$

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Research partially supported by the Ministerio de Educación y Cultura (Spain) grant PB96–1345)

This work is conected with the Parameter Free Paris–Friedman's Conjecture (see [1] Problem 34, and [2] page 398):

$$\mathbf{L}\Delta^-_{n+1} \iff \mathbf{I}\Delta^-_{n+1}$$

The relationships among the schemas of induction, minimalization and collection for formulas in the classes of the Arithmetic Hierarchy (Σ_n, Π_n) has been studied by J. Paris and L. A. Kirby (see [2] and [4]). The parameter free versions of these schemas have been studied by R. Kaye, J. Paris and C. Dimitracopoulos (see [3] and [5]).

Here we study the quantifier complexity of $\mathbf{L}\Delta^-_{n+1}$ and some relations between this theory and the class of the Π_{n+1} true sentences, $\mathbf{Th}_{\Pi_{n+1}}(\mathcal{N})$. In particular, we prove that

- **(a)** $\mathbf{L}\Delta^-_{n+1}$ is Σ_{n+2}–axiomatized but is not Π_{n+2}–axiomatizable.
- **(b)** $\mathbf{L}\Delta^-_{n+1}$ is not finitely axiomatizable.
- **(c)** $\mathbf{I}\Sigma_n$ is not an extension of $\mathbf{L}\Delta^-_{n+1}$. So, $\mathbf{L}\Delta^-_1$ is a proper extension of $\mathbf{I}\Delta_0$.
- **(d)** Let $\mathbf{T}$ be a theory such that $\mathbf{T}+\mathbf{exp}$ is consistent. If $\mathbf{Th}_{\Pi_2}(\mathbf{T})$ is an extension of $\mathbf{L}\Delta^-_1$ then $\mathbf{Th}_{\Pi_1}(\mathbf{T}+\mathbf{exp}) = \mathbf{Th}_{\Pi_1}(\mathcal{N})$.
- **(e)** For $n \geq 1$, $\mathbf{Th}_{\Pi_{n+1}}(\mathcal{N})$ is (up to equivalence) the unique theory Π_{n+1}–axiomatized that is an extension of $\mathbf{L}\Delta^-_{n+1}$.

Part **(e)** is proved in [5] for $\mathbf{I}\Pi^-_{n+1}$. Since, $\mathbf{I}\Pi^-_{n+1}$ is a proper extension of $\mathbf{L}\Delta^-_{n+1}$, the property given in **(e)** improves that result. Properties **(a)**-**(c)** and weak versions of **(d)** and **(e)** are also true for $\mathbf{I}\Delta^-_{n+1}$.

References

[1] Clote, P.; Krajícek, J. *Open Problems.* **Arithmetic, Proof Theory and Computational Complexity**, pages 1–19. Clarendon Press, Oxford, 1993.

[2] Hájek, P.; Pudlák, P. **Metamathematics of First Order Arithmetic**. Perspectives in Mathematical Logic, Springer Verlag, Berlin, 1993.

[3] Kaye, R. **Diophantine and Parameter-free Induction**. Ph.D. Thesis. University of Manchester, 1987.

[4] Kaye, R. **Models of Peano Arithmetic**. Clarendon Press, Oxford 1991.

[5] Kaye, R.; Paris, J.; Dimitracopoulos, C. *On parameter free induction schemas.* **The Journal of Symbolic Logic**. Vol. 43 (1988) 1082-1097.

(Research partially supported by the Ministerio de Educación y Cultura (Spain) grant PB96–1345)

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