Omitting Types and the Logic Action

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This research is on the omitting types property for Borel and Analytic sets in Polish G spaces, where G is any subgroup of the symmetric group S_∞ endowed with the pointwise topology.

S_∞ and its subgroups act on the space of countable structures for a given theory in a countable language. This space is universal for the S_∞-action.

We give a topological characterization of the notion of omitting types as a special requirement of orbit-separation or of boundedness of the orbital rank.

We say that a countable structure M strongly omits a non isolated orbit p if no suborbit q of p is locally dense; we show that this is equivalent to the following: q is not separable from p by a q dominating subtype s such that q is splittable with, respect to s. In general we prove that omitting types fails for a countable structure M whenever we can produce a G embedding between orbits that is not in the automorphism group of M.

We show that the existence of such a G-embedding depends on the presence of a group-stabilizer that is not a Jordan group.

Let M be a countable structure in a countable language; the main statement of this paper is the following:

Theorem 1: If $G \subset S_\infty$ is closed in the Baire space ω^ω , $G = Aut(M)$, and every M-type is a Jordan set, then the G omitting types holds for M.

As a corollary we have:

Corollary 2: The Scott sentence of M has no model of height ω, 1-elementary equivalent to M.

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