Quantum Logical Consequence

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Bruns and Kalmbach has described the lower part of the lattice of varieties of orthomodular lattices. Roddy has proved that the varieties generated by ortholattices MOn and $MO\omega$ varieties form an initial chain in lattice of varieties of modular ortholattices. In this paper we develop an algebraic technics to give a respective description of quasi-varieties. In particular we prove that: a) Let K be a semi-simple, congruence permutable variety of algebras. Every finite critical algebra $A \in K$ is isomorphic to direct product of finitely many pairwise non-isomorphic simple algebras from K. b) If $A = A_1 \times, ..., A_n$ be a product of finite, pairwise non-isomorphic finite algebras which not contain one-element subalgebras. then A is not critical if and only if there exists $i, j, k \in \{1, ..., n\}$ such that $A_i \hookrightarrow A_j$,$A_j \hookrightarrow A_k$, (the symbol $\hookrightarrow$ denotes "is isomorphic with a subalgebra of ") c) In particular, if the algebras A, B are non-isomorphic, finite, simple and not contain one-element subalgebras then $A \times B$ is critical.

The theorem above allows us to give a description of critical orthomodular lattices, to give then a complete description of upper part of the lattice of quantum deductive systems. In particular we get that there are 52 logical consequences in first three slices of the lattice of orthomodular logics and that there are $\frac{n(n+1)}{2}$ logical consequences stronger than the logic determined by MOn.

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