On Herbrand Theorem

Alexander V. Lyaletski

1st-order classical logic (with functional symbols and without equality) is considered. A new formulation of Herbrand theorem is suggested in such a form that the forms A, B, and C [1] can be obtained as consequences of the one. For this, an original notion of admissible substitution [2] is generalized in case of a set of arbitrary formulas.

Let S be a finite set of formulas. Herbrand quasi-universe $HQ(S)$ for S is the following minimal set of terms: (i) for any constant c from formulas belonging to S and for any variable x of a positive quantifier from formulas belonging to S it is true that c, $x \in HQ(S)$; (ii) if $t_1, ..., t_m \in S$ and a m-arity functional symbol f occurs in formulas from S, then $f(t_1,...,t_m) \in HQ(S)$.

THEOREM. A finite set S of formulas is unsatisfiable IFF there are a finite set S' of variants $F_1,...,F_n$ of formulas from S and a substitution σ of terms from $HQ(S')$ for all variables bounded by negative quantifiers from formulas belonging to S' such that: (1) formulas F_i and F_j have common bounded variables only if they are variants of a same formula from S and satisfy a certain condition; (2) σ is an admissible substitution for S'; (3) a formula $(M_1 * \sigma)$ & ... & $(M_n * \sigma)$ considered as a formula of the proposition calculus is false, where $M_1,...,M_n$ are obtained by elimination of all quantifiers in $F_1,...,F_n$, respectively, and $M_i * \sigma$ denotes the result of application of σ to M_i $(i = 1,...,n)$.

1. *Herbrand, J.*: Recherches sur la Theorie de la Demonstration. Travaux de la Societe des Sciences et de Lettres de Varsovie, Class III, Sciences Mathematiques et Physiques, 1930, volume **33**. 2. *Lyaletski, A.V.*: Variant of Herbrand Theorem for Formulas in Prefix Form (in Russian). Kibernetika, 1981, **1**, 112–116.

Address: Cybernetics Department, Taras Shevchenko University of Kyiv, 6, Glushkov avenue, 03127 Kyiv, Ukraine
E-mail: lav@tc.unicyb.kiev.ua