A measure-Theoretic Wagde Hierarchy

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We shall look at a relation $\leq_{\mathrm{BlW}}$ on sets of reals that is closely related to Wadge reducibility (*cf.* [4] and [1]). For two sets of reals A and B we say that $A \leq_{\mathrm{BlW}} B$ holds if and only if the Wadge game $G_{\mathrm{W}}(A, B)$ has probabilistic value 0 (*cf.* [2]). Of course, this is a weaker reducibility relation that Wadge reducibility in the sense that $A \leq_{\mathrm{W}} B$ implies $A \leq_{\mathrm{BlW}} B$. By a theorem of Neeman (*cf.* [3]), the two relations have to coincide under the assumption of Blackwell Determinacy throughout the $\mathbf{L}(\mathbb{R})$ hierarchy. It is an open question whether they are equal everywhere – this question is equivalent to "Does Blackwell Determinacy imply Wadge Determinacy?".

In this talk we shall show (assuming Blackwell Determinacy) that the relation $\leq_{\mathrm{BlW}}$ exhibits the very same structural properties as $\leq_{\mathrm{W}}$ under AD. They can only fail to be the same if $\leq_{\mathrm{W}}$ is not wellfounded, *i.e.*, if Wadge Determinacy is false.

[1] Alessandro **Andretta**, The Secret Lore of the Cabal, *monograph in preparation*

[2] Donald A. **Martin**, The Determinacy of Blackwell Games, Journal of Symbolic Logic 63 (1998), p. 1565–1581

[3] Donald A. **Martin**, Itay **Neeman**, The Strength of Blackwell Determinacy, *in preparation*

[4] Robert **van Wesep**, Wadge Degrees and Descriptive Set Theory, *in:* Alexander S. Kechris, Yiannis N. Moschovakis (*eds.*), Cabal Seminar 76–77, Proceedings, Caltech–UCLA Logic Seminar 1976–77, Berlin 1978 [Lecture Notes in Mathematics 689], p. 151–170

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