Simplicity Inside a Homogeneous Model

Olivier Lessmann

We work inside a homogeneous model M, i.e. any partial automorphism of M of size less than the size of M can be extended to an automorphism of M. The following are examples of abstract situations where each model has a homogeneous extension: first order theories, Robinson theories, e.c. models of a universal f.o. theory, theories in various logics (n-variable, infinitary logics) with amalgamation over sets. More concrete examples are Euclidean spaces. In a homogeneous model M, complete types correspond to orbits of the group of automorphisms, and Lascar strong types to orbits of strong automorphisms. We introduce a dependence relation inside M, which coincides with dividing when M is compact. We say that the homogeneous model M is *simple* if this dependence relation has local character and independent sequences exist for types with an unbounded number of realizations. When M is simple, we prove all the usual first order properties of dividing for this dependence relation – extension, symmetry, transitivity, amalgamation for Lascar strong types. The proofs are short and provide nice simplifications even in the first order case. When M is stable – i.e. M does not have the order property witnessed by indiscernibles – then each complete type has a bounded number of independent extensions. This work is joint with Steve Buechler.

Address: Dept of Mathematics (m/c 249), University of Illinois – Chicago, Chicago, IL 60607-7045 USA
E-mail: lessmann@uic.edu