Proof theory and computer science: from typographical engineering towards geometry

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What is a proof? Is it a number (Gödel), a text (Hilbert), or a tree (Gentzen)?

- *Numbers* are old and respectable mathematical objects, but Gödel's numbering is arbitrary. It is just a tool for the proof of the incompleteness theorem.
- *Texts* are closer to the material reality of mathematical proofs, but they are more *typographical* objects than mathematical ones. Furthermore, there are many arbitrary choices in the textual representation.
- The structure of proofs is better shown by *trees*, and this representation is used indeed by systems for interactive proof search, but trees are still not the good objects !

Linear logic suggests a more geometrical viewpoint, with the notion of *proof net* and its avatars, *interaction nets* and *geometry of interaction*. In a related area, namely *symbolic computation* and *rewriting*, geometrical notions also appear, such as *Penrose diagrams* and *homological invariants*. A symbiosis between logic and geometry could be as fruitful as the one which already exists between algebra and geometry.

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