Integer Division and Second Order Weak Bounded Arithmetic

Satoru Kuroda

The theory T^{pol} is a second order theory whose $\Delta_1^{1,w}$ definable predicates (intuitively, provably $NP \cap$ *co-NP* predicates) are exactly those decidable in polynomial time. This theory, however, has a very poor ability in defining functions. Namely, Takeuti [Tak] proved that T^{pol} cannot define the function $\lfloor a/b \rfloor$.

In the present talk, we will improve this result in the following way: let T^{pol+} be the theory T^{pol} extended by the function $MSP(x,y) = \lfloor x/2^{|y|} \rfloor$. Then we have

Theorem 1 *For $k \in \omega$, $T^{pol+} \vdash \forall x \exists y (y = \lfloor x/k \rfloor$ if and only if k is not a power of 2.*

Contrary to the theorem of Takeuti, our proof is model-theoretical and makes use of the method of length initial substructures developed by Johannsen [Joh].

[CT] P. Clote and G. Takeuti. *Bounded arithmetic for NC, AlogTIME, L and NL*. Annals of Pure and Applied Logic. 56 (1992) 73–117.

[Joh] J. Johannsen. *A remark on independence results for sharply bounded arithmetic*. Mathematical Logic Quarterly. 44 (1998) 569–571.

[Tak] G. Takeuti. *A second order version of S_2^i and U_2^1*. JSL. **56** (1991) 1038–1063.

Address: Gumma Prefectural Women's University, 1395-1 Kaminote, Tamamura-machi, Sawagun, Gunma, 370-1193, Japan
E-mail: `satoru@@math.human.nagoya-u.ac.jp`