On Minimally Unsatisfiable Conjunctive Normal Forms

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We give an overview on recent progress towards an understanding of the structure of *minimally unsatisfiable clause-sets*, focussing on the notion of *deficiency* $\delta(F) = c(F) - n(F)$ ([4]), where $c(F)$ is the number of clauses and $n(F)$ is the number of variables, and considering also generalisations like *lean clause-sets* ([5]), which are clause-sets without non-trivial *autarkies*.

Starting with [1], where $\delta(F) \geq 1$ for minimally unsatisfiable F has been shown, and the class of "saturated" minimally unsatisfiable F with $\delta(F) = 1$ has been characterised, one central problem has been the *classification* of minimally unsatisfiable clause-sets with small deficiency. In [3] the class of all minimally unsatisfiable F with $\delta(F) = 1$ is characterised, and in [2] an upper bound for tree-like resolution depending on the deficiency is given. In [6] for all k the decision problem for minimally unsatisfiable F with $\delta(F) \leq k$ has been shown to be in *polynomial time* (see also [7] for further developments).

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[3] Gennady Davydov, Inna Davydova, and Hans Kleine Büning. An efficient algorithm for the minimal unsatisfiability problem for a subclass of CNF. *Annals of Mathematics and Artificial Intelligence*, 23:229–245, 1998.

[4] John Franco and Allen Van Gelder. A perspective on certain polynomial time solvable classes of satisfiability. To appear in Discrete Applied Mathematics, 2000.

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