Honest and Semi-honest Subrecursive Degrees

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Let $T_0 \preceq T_1$ denote that each computable function, which is provable total in the first order theory T_0, is also provable total in the first order theory T_1. The relation $\preceq$ induces a degree structure on the sound finite Π_2 extensions of EA (Elementary Arithmetic). [1] is devoted to the study of this structure. However, we do not study the structure directly. Rather we define an isomorphic subrecursive degree structure $\langle \leq, \mathcal{S} \rangle$, and then we study $\langle \leq, \mathcal{S} \rangle$ by subrecursive and computability-theoretic means. Furthermore, we introduce and investigate some operators on the degrees of $\langle \leq, \mathcal{S} \rangle$. These operators correspond to inference rules in formal arithmetic. One operator (the collection operator) corresponds to the Σ_1 collection rule. Another operator (the jump operator) corresponds to the Σ_1 induction rule.

The structure $\langle \leq, \mathcal{S} \rangle$ is called the structure of semi-honest degrees. The structure of honest degrees is a strict substructure of $\langle \leq, \mathcal{S} \rangle$. In [2] and [3] we study the jump operator on the honest degrees.

[1] Kristiansen, L. *Subrecursive Degrees and Fragments of Peano Arithmetic* (Accepted for publication in Archive for Mathematical Logic)

[2] Kristiansen, L. *Low_n, $high_n$, and intermediate subrecursive degrees.* In Calude and Dinneen (Eds.) Combinatorics, computation and logic. (Proceedings) Springer, Singapore 1999, 286-300.

[3] Kristiansen, L. *A jump operator on honest subrecursive degrees.* Archive for Mathematical Logic 37 (1998), 105-125.

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