Infinitary Axiomatizabiliy of Slender and Cotorsion-free Groups

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This talk contains some results on the axiomatizability in higher-order logic of the classes of slender and cotorsion-free infinite abelian groups.

Recall that the *Baer-Specker group* $\mathbb{Z}^\omega$ is the product of countably many copies of $\mathbb{Z}$ (the integers). A torsion-free group G is *slender* if whenever $h : \mathbb{Z}^\omega \to G$ is a homomorphism, then $h(e_n) = 0$ for all but finitely many $n \in \mathbb{N}$, where e_n is the standard n-th unit vector $(0, \ldots, 0, 1, 0, \ldots) \in \mathbb{Z}^\omega$. A group A is *cotorsion* if $\mathrm{Ext}(J, A) = 0$ for all torsion-free groups J. A group G is *cotorsion-free* if G does not contain any non-zero subgroup which is cotorsion.

In 1974, Paul Eklof used infinitary logic to generalize some classical theorems of infinite abelian group theory. He characterized the strongly $\aleph_1$-free groups as exactly those abelian groups which are $L_{\infty\omega_1}$-equivalent to free abelian groups, used his criterion to deduce that the class of free abelian groups is not $L_{\infty\omega_1}$-definable, and showed that the Baer-Specker group $\mathbb{Z}^\omega$ is not $L_{\infty\omega_1}$-equivalent to a free abelian group, strengthening a theorem of Baer that $\mathbb{Z}^\omega$ is not free. The present talk continues in the tradition allying infinite abelian group theory with infinitary logic. Its main results are the following:

Theorem 1 *The class of slender abelian groups is axiomatizable in the infinitary logic $L_{\infty\omega_1}$.*

Corollary 2 *The Baer-Specker group $\mathbb{Z}^\omega$ is not $L_{\infty\omega_1}$-equivalent to a slender group.*

Corollary 3 *The class of cotorsion-free groups is $L_{\infty\omega}$-definable.*

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