Generic Automorphisms and the DMP

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This is a joint work with Anand Pillay. Given a complete theory T with quantifier elimination in a language L, we consider the (incomplete) theory $T \cup \{$"σ is an L-automorphism"$\}$ in the language $L \cup \{\sigma\}$ and denote its model companion by TA if it exists. Similarly, we denote the model companions for T with two automorphisms and for T with countably many automorphisms by TA_2 and TA_ω respectively.

When T is the theory ACF of algebraically closed fields, TA exists and called $ACFA$. One axiomatization of $ACFA$ is based on the fact that ACF has the DMP (definable multiplicity property).

Theorem 1. Let T be a strongly minimal theory with quantifier elimination. The following are equivalent: (1) TA_2 exists; (2) TA_ω exists; (3) T has the DMP.

Chatzidakis pointed out to us that (1) implies (2).

Theorem 2. Let T be a strongly minimal theory with quantifier elimination which is a finite cover of a strongly minimal theory which does have the DMP. Then TA exists if and only if T has the DMP.

In general, there are many unstable theories T with no TA. For example, if T is unstable without independence property then there is no TA. Also, if T has the fcp then T_σ has no model completion ([1]). As far as I know, every known theory T which has TA is stable.

[1] H. Kikyo, Model companions of theories with an automorphism, MSRI Preprint #1998-039 (1998) (to appear in Journal of Symbolic Logic).

[2] H. Kikyo and A. Pillay, The definable multiplicity property and generic automorphisms, to appear in Annals of Pure and Applied Logic.

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