Relation Between T-Degrees and Σ-Degrees of a Model

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Consider those models that consist of a countable universe and a finite number of predicates and functions. Yu.L. Ershov has introduced in [1] the notion of Σ-definability of a model $\mathfrak{A}$ in the admissible set $HF(\mathfrak{B})$ over a model $\mathfrak{B}$. We say that a model $\mathfrak{A}$ is Σ-reducible to a model $\mathfrak{B}$ and write $\mathfrak{A} \leq_\Sigma \mathfrak{B}$ (or simply $\mathfrak{A} \leq \mathfrak{B}$), if a model $\mathfrak{A}$ is Σ-definable in $HF(\mathfrak{B})$. From results of Ershov [2, p. 179] we obtain that the relation $\leq$ on the class K of all models is a preorder with the least elements. If we use $\leq_\Sigma$ to generate a equivalence $\equiv_\Sigma$ on class K, where $\mathfrak{A} \equiv \mathfrak{B}$ iff $\mathfrak{A} \leq \mathfrak{B}$ and $\mathfrak{B} \leq \mathfrak{A}$, then the model $L_\Sigma =< K/_\equiv, \leq>$ is a partially ordered set. The class $\overline{\mathfrak{A}} \in L_\Sigma$ containing a model $\mathfrak{A}$ is called the Σ-degree of the model $\mathfrak{A}$. Let $\mathfrak{A} \oplus \mathfrak{B}$ be the disjoint union of models $\mathfrak{A}$ and $\mathfrak{B}$. Then the class $\overline{\mathfrak{A} \oplus \mathfrak{B}}$ is the least upper bound of classes $\overline{\mathfrak{A}}$ and $\overline{\mathfrak{B}}$, i.e. we have the following

PROPOSITION 1. The model L_Σ is an upper semilattice with the least element.

Remark. From results of S.S. Goncharov [3] we obtain that a Σ-degree of $\overline{\mathfrak{A} \times \mathfrak{B}}$ is not always equal to $\overline{\mathfrak{A} \oplus \mathfrak{B}}$.

From results of [4,6] we obtain

PROPOSITION 2. If models $\mathfrak{A}$ and $\mathfrak{B}$ has degrees a and b respectively and $\mathfrak{A} \leq_\Sigma \mathfrak{B}$ then $a \leq_T b$.

PROPOSITION 3. There are models $\mathfrak{A}$ and $\mathfrak{B}$ whose universe is the set of natural numbers and degrees of their open diagrams are equal, but $\mathfrak{B}$ is not Σ-definable in $HF(\mathfrak{A})$.

Assume that a map φ from a set of T-degrees to L_Σ is defined by $\varphi(a) = \overline{\mathfrak{A}}$, where $\mathfrak{A}$ is a finitely generated model of degree a. Then we obtain

THEOREM. The map φ is an embedding of the upper semilattice of T-degrees into the upper semilattice of Σ-degrees.

References

1. Ershov Yu.L.(1985) Σ-Definability in admissible sets, Dokl. Akad. Nayk SSSR, v.285, no. 4, pp. 792–795.

2. Ershov Yu.L.(1996) Definability and computability, Plenum, New-York.

3. Goncharov S.S.(1979) Constructivization of Cartesian powers of models, 15-th All-Union algebraic conference, Krasnoyarsk.

4. Khisamiev A.N.(1998) On definability of a model in a hereditary finite admissible set, Generalized definability and computability, Vychislitel'nye sistemy, no. 161, pp. 15–20.

5. Khisamiev A.N.(1997) Numbering and definability in the hereditary finite superstructure of a model, Siberian Advances in Mathematics, v. 7, no. 3, pp. 63–74.

6. Khisamiev A.N.(1998) Strong Δ-definability of a model in an admisible set, Siberian Math. J., vol. 39, no. 1, 1998, pp. 168–176.

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