On Universality Properties of Reduced Products

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In the early 1970's S.Tennenbaum proved that all countable models of $PA^- + \forall_1 - Th(\mathbb{N})$ are embeddable into the reduced product $\mathbb{N}^\omega/\mathcal{F}$, where $\mathcal{F}$ is the cofinite filter.

Theorem 1 *If M is a model of $PA^- + \forall_1 - Th(\mathbb{N})$, and $|M| = \aleph_1$, then M is embeddable into $\mathbb{N}^\omega/\mathcal{F}$.*

Theorem 2 *If M is a model of $PA^- + \forall_1 - Th(\mathbb{N})$, $|M| = \lambda^+$, $(\aleph_1, \aleph_0) \to (\lambda^+, \lambda)$ and $\mathcal{F}$ is a regular filter on λ, then M is embeddable into $\mathbb{N}^\lambda/\mathcal{F}$*

Theorem 3 *If $\mathcal{F}$ is a regular ultrafilter on λ, $(\aleph_1, \aleph_0) \to (\lambda^+, \lambda)$ and M is infinite, then $M^\lambda/\mathcal{F}$ is λ^{++}-universal. (This answers Open Question 19 of Chang & Keisler, Model Theory, North-Holland).*

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