Hilbert's Paradox

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In 1903 Gottlob Frege sent a complimentary copy of the second volume of the *Grundgesetze der Arithmetik* to the Göttingen mathematician David Hilbert, containing in the postscript the description of Russell's Paradox and Frege's admission that this paradox can be formulated in the system of the Grundgesetze. In his response Hilbert declared that the paradox described had been known in Göttingen for a long time. He himself had found other, even more convincing examples four to five years ago, and after having informed Zermelo the latter found the one mentioned by Frege three to four years before.

It is well known that Zermelo indeed discovered a set-theoretical paradox in Cantor's theory, independently of Russell. But what were these contradictions Hilbert claimed to have found around 1898/1899? Hilbert never published a contradiction.

One of the author's was able to discover a paradox which is most likely the one Hilbert referred to in his letter to Frege. It is presented in an unpublished lecture course of Hilbert's delivered in the summer term of 1905 on *Logische Principien des mathematischen Denkens*. There Hilbert discusses the paradoxes of set theory mentioning Zermelo's paradox and a contradiction of "purely mathematical nature" which was never published, as Hilbert stressed, but known to set-theorists, especially to Georg Cantor.

In the historical part of the lecture the story of Hilbert's Paradox will be told and it will be shown that Hilbert's axiomatic program was indeed affected by the paradoxes, contrary to the standard view. In the systematical part of the lecture Hilbert's Paradox will be described and reconstructed using modern tools. It will be shown that it is closer to Cantor's Paradox than to Russell's.

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