Actions of Automorphism Groups on Real Trees

Alexandre Ivanov

Let G be an infinite group acting on an $\mathbf{R}$-tree by homeomorphisms. The action is called *non-nesting* if no $g \in G$ maps an arc properly into itself [G.Levitt, Non-nesting actions on real trees, Bull. London Math. Soc. 30(1998), 46 – 54]. In this abstract we always assume that G is a closed subgroup of $Sym(\omega)$ (*the automorphism group of a structure M on ω*) and its actions are non-nesting with closed stabilizers (the stabilizer is closed when the action is Borel).

A structure N *realizes orthogonal copies* if for any tuple $\bar{b} \subset N$ and any finite $A \subset N$ there exists $\bar{b}' \subset N$ of the type $tp(\bar{b})$ such that the restriction map $tp(\bar{a}/\bar{b}') \to tp(\bar{a})$, with $\bar{a} \subseteq A$, is one-to-one [A.Ivanov, Finite covers, cohomology and homogeneous structures, Proc. London Math. Soc., 78(1999), 1 – 28].

Theorem 1. *Let M be a countable ω-homogeneous sructure realising orthogonal copies and $G = Aut(M)$. Then for any non-nesting action of G on a real tree with clopen stabilizer any $g \in G$ fixes a point.*

The theorem implies that if M additionally has the small index property then each $g \in G$ fixes a point with respect to any action on a $\mathbf{Z}$-tree. This implies that the automorphism group of an arithmeticaly saturated model of PA has property FA defined in [J.P.Serre. Trees. Springer-Verlag 1980].

Theorem 2. *Let $A(\mathbf{Q})$ be the group of all order-preserving permutations of $(\mathbf{Q}, <)$. Let $G \in \{A(\mathbf{Q}), Sym(\omega)\}$. Then for any non-nesting action of G on a real tree each element of G fixes a point.*

Let A be a countable periodic abelian group and $H(A)$ be the universal locally finite extension of A [K.Hickin, Universal locally finite central extensions of groups, Proc. London Math. Soc., 52(1986), 53 – 72].

Theorem 3. *For any isometric action of $G = Aut(H(A))$ on a real tree the group G acts by translations on a G-invariant geodesic line or any element of G fixes a point. The group G has property FA if and only if for no p the p-component of the divisible part of A consists of finitely many quasicyclic groups.*

Address: Institute of Mathematics, Wrocław University, pl.Grunwaldzki 2/4, 50-384 Wrocław, Poland
E-mail: `ivanov@math.uni.wroc.pl`