A Dichotomy for the Asymptotic Geometry of Sequences

José Iovino

A central concept in Banach space geometry is that of spreading model. (See [2, 3] and [1].) Every basic sequence in a Banach space has a subsequence that generates a Banach space called a spreading model of the sequence, and the spreading models generated by a sequence are the standard to analyze is asymptotic geometry.

A central question is to identify under what conditions a spreading contains either of the classical sequence spaces ℓ_p and c_0. To date, most of the results in this direction have been negative. (See, for example, [4] and [5]).

We will state the following dichotomy.

Theorem 1 *Every basic sequence in a Banach space X has a normalized block basis (x_n) which satisfies one and only one of the following conditions.*

1. *(x_n) is cofinally isometric over X to either c_0 or ℓ_p, for some p with $1 \leq p < \infty$.*
2. *(x_n) has the largest possible number of spreading models.*

The proof is based on constructions from model theory. Specifically, we use the fact that there is an intimate connection between whether a spreading model contains ℓ_p or c_0 and whether the spreading model has a type that is stable in the sense of model theory. The preceding theorem is a counterpart of Shelah's landmark result that an unstable theory has the maximum number of models.

[1] B. Beauzamy and J.-T. Lapresté. *Modèles étalés des espaces de Banach.* Travaux en Cours. [Works in Progress]. Hermann, Paris, 1984.

[2] A. Brunel. Espaces associés à une suite bornée dans un espace de Banach. page 23, 1974.

[3] A. Brunel and L. Sucheston. On B-convex Banach spaces. *Math. Systems Theory*, 7(4):294–299, 1974.

[4] E. Odell and Th. Schlumprecht. On the richness of the set of p's in Krivine's theorem. In *Geometric aspects of functional analysis (Israel, 1992–1994)*, volume 77 of *Oper. Theory Adv. Appl.*, pages 177–198. Birkhäuser, Basel, 1995.

[5] E. Odell and Th. Schlumprecht. A problem on spreading models. *J. Funct. Anal.*, 153(2):249–261, 1998.

Address: Division of Mathematics and Statistics, The University of Texas at San Antonio, San Antonio, TX 78249-0664, USA
E-mail: `iovino@math.utsa.edu`