Paradoxes for Tense Logic

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The difficulties with formalizing the intensional notions *necessity, knowability* and *omniscience*, and *rational belief* are well-known. If these notions are applied to (codes of) sentences, then from apparently weak and uncontroversial logical principles governing these notions outright contradictions can be derived.

Tense logic is one of the best understood and most extensively developed branches of intensional logic. In tense logic, the temporal notions past and future are formalized as sentential operators rather than as predicates. The question therefore arises whether the notions that are investigated in tense logic can be consistently formalized as predicates.

In this paper it is shown that the answer to this question is negative. Consider the language of arithmetic plus two additional primitive unary predicates P and F (past and future). Let K_t^* be the theory that is obtained by adding to Robinson Arithmetic the axioms and rules of the minimal tense logic K_t (except that past and future are now treated as predicates). Then an application of Gödel's diagonal lemma yield the following paradoxical result:

Theorem 1 $K_t^* \vdash \neg P(\overline{\mathtt{T}}) \wedge) P \neg (\overline{\mathtt{T}})$

If in addition the principle $K_t^* + \neg P(\overline{\neg\varphi}) \rightarrow P(\overline{\varphi})$, which says that there exists a moment in the past, is assumed, an outright contradiction follows:

Theorem 2 $K_t^* + \neg P(\overline{\neg\varphi}) \rightarrow P(\overline{\varphi}) \vdash \bot$.

These two paradoxes result from the *logical interplay* between the notions of past and future. In this sense, they are different from the paradoxes of Kaplan, Montague and Thomason.

[KM] Kaplan, D. & Montague, R. A Paradox Regained. Notre Dame Journal of Formal Logic 1(1960), 79-90.

[M] Montague, R. Treatments of Modality with Corollaries on Reflexion Principles and Finite Axiomatizability. Acta Philosophica Fennica 16(1963), 153-167.

[T] Thomason, R. Note on Syntactical Treatments of Modality. Synthese 44(1980), 391-395.

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