A Contribution on t-norm Based Logics

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A standard algebra (for fuzzy propositional calculus) is a structure given by a continuous t-norm on the real unit interval. Continuous t-norms are taken as truth functions of the conjunction, whereas their residua provide truth functions of the implication. A detailed treatment of fuzzy propositional calculus, on which this contribution is based, will be found in [1], where the basic logic (BL) is proposed; [2] proves that BL is complete w.r.t. propositional formulas which are tautologies of all standard algebras.

The representation theorem for continuous t-norms says that any standard algebra is an ordered sum of isomorphic copies of the standard algebras given by Łukasiewicz's, Gödel's, and product t-norms (we denote these standard algebras $[0,1]_{\mathrm{L}}$, $[0,1]_{\mathrm{G}}$, and $[0,1]_{\Pi}$ respectively). The three schematic extensions of BL corresponding to these standard algebras are investigated in [1].

Our aim is to clarify some properties of other standard algebras. It is not difficult to observe that the tautologies of a standard algebra A (an ordered sum of $[0,1]_{\mathrm{L}}$'s, $[0,1]_{\mathrm{G}}$'s, and $[0,1]_{\Pi}$'s) are included in the tautologies of a standard algebra obtained from A by deleting some members of the ordered sum (provided that no isomorphic copy of $[0,1]_{\mathrm{L}}$ is thus shifted to the very beginning of the sum). This observation imposes some necessary inclusions of tautologies.

We propose formulas which characterize, up to isomorphisms and the abovementioned inclusions of tautologies, particular cases of standard algebras, e.g., simple ordered sums like $[0,1]_{\mathrm{L}\oplus\mathrm{G}}$, $[0,1]_{\mathrm{G}\oplus\Pi}$ etc. Completeness for any such schematic extension of BL w.r.t. the corresponding standard algebra follows from the representation theorem for linearly ordered BL-algebras presented in [2] (a generalization of the representation theorem for continuous t-norms), and a completeness theorem for linearly ordered BL-algebras from [1].

The hierarchy of inclusions does not collapse trivially to BL: it can be shown that tautologies of standard algebras which are ordered sums of $[0,1]_{\mathrm{G}}$'s and $[0,1]_{\Pi}$'s only, form an infinite chain of sharp inclusions, ordered by the number of copies of $[0,1]_{\Pi}$'s. We cannot present a similar result for sums of $[0,1]_{\mathrm{L}}$'s so far.

References [1] Hájek P.: *Metamathematics of Fuzzy Logic.* Kluwer Academic Publishers, 1998.

[2] Cignoli R., Esteva F., Godo L., Torrens A.: *Basic fuzzy logic is the logic of continuous t-norms and their residua.* To appear in Archive for Mathematical Logic.

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