Properties of Monadic Fuzzy Predicate Logics

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Ł∀, G∀, Π∀ denote the three most important particular t-norm predicate logics: Łukasiewicz, Gödel and product logic. Their set of thruth values is the real unit interval and the semantics is fully determined by the corresponding continuous t-norm as the truth function of conjunction. (See [1] for metamathematics of t-norm based logics.) Each of them has two monadic counterparts: mon-Ł∀ is Ł∀ with only unary predicates and mon_1-Ł∀ is Ł∀ with only unary predicates and only one object variable. Similarly for G∀ and Π∀. For each logic, TAUT is the set of all tautologies (formulas true, i.e. having truth value 1 in each interpretation), f-TAUT is the set of all finite tautologies (true in each finite interpretation) and similarly for SAT, f-SAT (true in at least one [finite] interpretation). Finite model properties: FMP1 is TAUT = f-TAUT, FMP2 is SAT = f-SAT. What is the arithmetical complexity of these sets? What finite model properties hold? We summarize the results (some of them are found in [1], [2], [3]).

Theorem (1) For Ł∀, TAUT is Π_2-complete, f-TAUT is Π_1-complete, both FMP1 and FMP2 is false. For mon-Ł∀, FMP1 is true but FMP2 not; TAUT = f-TAUT is in Π_1. For mon_1Ł∀, both FMP1 and FMP2 is true, TAUT = f-TAUT is Δ_1.

(2) Both FMP1 and FMP2 are false for mon_1-G∀ (hence for mon-G∀ and G∀). For G∀, TAUT is Σ_1-complete and f-TAUT is Π_1-complete.

(3) For Π∀, TAUT is Π_2-hard and f-TAUT is Π_1-complete. Both FMP1 and FMP2 fails for mon_1-Π∀.

References [1] P. Hájek: Metamathematics of fuzzy logic, Kluwer 1998.

[2] P. Hájek: Trakhtenbrot theorem and fuzzy logic. In: (Gottlob et al., ed.) Computer Science Logic, Springer Verlag 1999, p. 1-8.

[3] M. Ragaz: Die Unentscheidbarkeit der einstelligen unendlichwertigen Prädikatenlogik. Arch. Math. Logik 23 (1983) 129-139.

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