Some Functional Schemes in Arithmetic

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Peano Arithmetic, **PA**, can be described through the classical axioms schemes of induction (**I**), minimization (**L**), collection (**B**) and strong collection (**S**) for the hierarchy of Σ_n and Π_n-formulas. The relationships between the schemes above for that hierarchy were established by J. Paris and L. Kirby (1977) [1], for induction, minimization and collection. And by P. Clote (1983) for strong collection.

In this work, we introduce some new functional schemes about the existence of a maximum element as follows: Given a formula $\Theta(x, y)$ of the language of Arithmetic, the axiom $\mathbf{F^*}_\Theta$ states that if Θ defines a function with nonempty and bounded domain, then its range has a maximum element. On the other hand, the axiom $\mathbf{FT^*}_\Theta$ states the same, but with Θ defining a total function. If $\Gamma \subseteq \mathbf{Form}(\mathcal{L})$, then we can consider the theories $\mathbf{F}^*\Gamma$ and $\mathbf{FT}^*\Gamma$.

The main result of this work is the following:

Theorem 1 *For every $n \in \omega$:*

$$\begin{array}{ccccccc} \mathbf{S}\Sigma_{n+1} & \Longleftrightarrow & \mathbf{F}^*\Sigma_{n+1} & \Longleftrightarrow & \mathbf{F}^*\Pi_n & \Longleftrightarrow & \mathbf{S}\Pi_n \\ \mathbf{B}\Sigma_{n+1} & \Longleftrightarrow & \mathbf{FT}^*\Sigma_{n+1} & \Longleftrightarrow & \mathbf{FT}^*\Pi_n & \Longleftrightarrow & \mathbf{B}\Pi_n \end{array}$$

Finally, we state the equivalence between the scheme $\mathbf{FT}^*$ and the scheme of minimization for Δ_{n+1}-formulas, what allows us for a new "functional" formulation of the Friedmann-Paris conjecture, in relation to the equivalence between the axioms schemes of induction and minimization for Δ_{n+1}-formulas.

[1] Paris, J., Kirby, L., *Σ_n-collection schemas in Arithmetic*, **Logic Colloquium '77**, (1978), 199-209

[2] Pérez Jiménez, M.J., *Esquemas del máximo en la Aritmética*, **Ph.D, Thesis**, (1992), Universidad de Sevilla

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