On Ajdukiewicz's Categorical Analysis of Quantifiers

Béatrice Godart-Wendling & Pierre Joray

The first time that a solution was given to the categorial analysis of formal quantifiers was in the second part of K. Ajdukiewicz's famous paper of 1935: "Die syntaktische Konnexitt". Inspired by S. Lesniewski's logical inquiries, this solution allows both to specify and to complete the theory of semantic categories which was still problematic on this point. Let us mention, as a reminder, that Lesniewski did not attach any categorial index to the quantifiers in his formal language. Such a characterization would have led Lesniewski to put his high-order quantification among the functors of his calculus and call into question the unity of quantification, imposing a different index on the quantifier each time it has to bind a variable of a different semantic category.

Ajdukiewicz's treatment consists then to introduce an essential distinction between what he called "functor" and "operator". This allows him to indicate accurately, in the categorial formalism, the notion of scope and the binding role of quantifiers. Adding to his categorial notation the use of a vertical dash, left to the quantifiers fractional indexes, Ajdukiewicz manages to represent in the formalism both the shared and the distinctive properties of functors and operators. Just like any functor, a quantifier can form a composite coherent whole with the argument to which it applies. However, as being an operator, a quantifier can never be one of the arguments of a functor of an higher order. The vertical dash takes this difference into account. It introduces a new condition into the calculus concerning the order of application of Ajdukiewicz's unique categorial rule. Extending, moreover, this analysis to Whitehead and Russell's circumflex operator $\hat{}$, Ajdukiewicz manages then to show that every quantifier can be specifically described as a combination including a functor and a pure abstraction operator.

In this paper, we will show the way Ajdukiewicz's formalization constitutes a valuable generalization of Lesniewski's theory of semantic categories. This study will also lead us to specify how Ajdukiewicz's solution allows an homogeneous treatment of all the formal quantifiers.

AJDUKIEWICZ K. (1935). "Die syntaktische Konnexitt". Studia Philosophica 1. 1-27. (Eng. tr. in Mc Call S. Polish Logic, 1920-1939. Oxford: Clarendon. 1967. 207-231).

LESNIEWSKI S. (1992). Collected Works. Surma S., Srzednicki J. T., Barnett D.I. (eds). Varsaw/Dordrecht: Polish scientific Pub./Kluwer.

WHITEHEAD A. N., RUSSELL B. (1927). Principia Mathematica. (2nd ed.). Cambridge University Press.

Addresses

Béatrice Godart-Wendling : CNRS, Laboratoire d'histoire des théories linguistiques Université Paris 7; 2, Place Jussieu, F-75005 Paris, France

E-mail: `godart@ccr.jussieu.fr`

Pierre Joray : Séminaire de Logique, Université de Neuchâtel, Louis-Agassiz 1, CH-2000 Neuchâtel, Switzerland

E-mail: `pierre.joray@lettres.unine.ch`