Minimal Degrees Below High Degrees

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A degree $\mathbf{a} \leq \mathbf{0}'$ is *high* if $\mathbf{a}' = \mathbf{0}''$, and a degree $\mathbf{a}$ is *generalized high* if $\mathbf{a}' = (\mathbf{a} \cup \mathbf{0}')'$. We note that generalized high degrees and high degrees coincide on the degrees below $\mathbf{0}'$. A degree $\mathbf{m}$ is *minimal* if $\mathbf{m} > \mathbf{0}$ and there is no degree $\mathbf{d}$ such that $\mathbf{0} < \mathbf{d} < \mathbf{m}$. Minimal degrees do indeed exist, and it has been shown that there are $2^{\aleph_0}$ minimal degrees. Every high degree bounds a minimal degree, and no high degree is itself minimal. In fact every generalized high degree bounds a minimal degree. There are results of Cooper and Epstein, and of Li and Yang, showing that there are elementary differences in the way individual members of familiar classes of degrees known to bound minimal degrees do so. We investigate possible distributions of minimal degrees below high degrees.

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